Sem-II / Applied Maths-II / 18-11-14

(REVISED COURSE) QP Code : **11859**

(3 Hours) **Total Marks : 80**

N. B. : (1) Question No.1 is **compulsory.**
(2) Attempt **any three** questions of the remaining **five.**
(3) **figures** to the right indicate **full** marks.

1. (a) Evaluate $\int_0^2 x^4 (8-x^3)^{-1/3}\, dx$ 3

(b) Solve $\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y=0$ 3

(c) Prove that $E = 1+\Delta = e^{4D}$ 3

(d) Solve $\left[x\sqrt{x^2+y^2}-y\right]dx+[y\sqrt{x^2+y^2}-x]\,dy=0$ 3

(e) Change to polar coordinates and evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$ 4

(f) Evaluate $\int_0^1\int_0^x e^{x+y}\,dydx.$ 4

2. (a) Solve $\frac{dy}{dx}+x\sin 2y = x^3\cos^2 y$ 6

(b) Change the order of integration and evaluate $\int_0^a\int_{y^2/a}^{y} \frac{y}{(a-x)\sqrt{ax-y^2}}dxdy$ 6

(c) Prove that $\int_0^\infty \cos\lambda x\left(e^{-ax}-e^{-bx}\right)dx=\frac{1}{2}\log\left(\frac{b^2+\lambda^2}{a^2+\lambda^2}\right)$, a>0, b>0 using DUIS rule 8

3. (a) Evaluate $\iiint \frac{dx\,dy\,dz}{x^2+y^2+z^2}$ throughout the volume of the sphere $x^2+y^2+z^2=a^2$ 6

(b) Find the area common to the cardiods r = a (1+cos θ) and r = a (1- cos θ). 6

(c) Apply the method of variation of parameters to solve 8

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{2x}\sec^2 x$$

4. (a) Find the length of one arc of the cycloid x = a (θ - sin θ) and y = a (1+ cos θ) 6

(b) Solve $\frac{d^2y}{dx^2}+2y=x^2e^{3x}+e^x\cos x$ 6

(c) Apply Runge- kutla method of fourth order to find an approximate value of y at $x = 1.2$ if $\frac{dy}{dx} = x^2 + y^2$, given that $y = 1.5$ when $x = 1$ choosing $h = 0.1$ 8

5. (a) Solve $[xy^2 - e^{1/x^3}]\,dx - yx^2\,dy = 0$ 6

(b) If y satisfies the equation $\frac{dy}{dx} = x^2 y - 1$ and with $y = 1$ when $x = 0$, using Taylor's series method for y about $x = 0$, find y when $x = 0.1$ and $x = 0.2$ 6

(c) Compute the value of the definite integral $\int_{-1}^{1} \frac{dx}{1+x^2}$ by using 8

(i) Trapezoidal rule

(ii) Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule

(iii) Simpson's $\left(\frac{3}{8}\right)^{th}$ rule. Compare result with exact values.

6. (a) A radial displacement 'u' in rotating a disc at a distance 'r' from the axis in given by $\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} + kr = 0$. Find the displacement given $u = 0$ when $r = 0$ and $r = a$ 6

(b) Evaluate $\iint x^2\,dxdy$ over the region bounded by $xy = a^2$, $x = 2a$, $y = 0$ and $y = x$ in the first quadrant. 6

(c) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ 8
