

(REVISED COURSE)

QP Code : 1037

(3 Hours)

[Total Marks : 80]

N.B. (1) Question no. 1 is compulsory.

(2) Solve any three questions from the remaining six questions

(3) Each questions carry equal marks.

1. (a) Evaluate $\int_0^{\infty} \frac{x^4}{4^x} dx$ 3
- (b) Find P.I. of $(D^2 - 4D + 4)y = e^{2x} \cos 2x$ 3
- (c) Show that $\nabla = 1 - E^{-1}$ 3
- (d) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy dx}{1+x^2+y^2}$ 3
- (e) Solve $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$ 4
- (f) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$ by changing to polar co-ordinates 4
2. (a) Solve $y^4 dx = (x^{-\frac{3}{4}} - y^3 x) dy$ 6
- (b) Change the order of integration and evaluate $\int_0^1 \int_x^{\frac{1}{2}} \frac{y}{(1+xy)^2(1+y^2)} dy dx$ 6
- (c) (1) P.T. $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{1}{a^n b^n} \beta(m, n)$ 4
- (2) P.T. $\int_0^{\infty} \frac{\log(i+ax^2)}{x^2} dx = \pi\sqrt{a}$, where $a > 0$ 4
3. (a) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x-\log y} e^{x+y+z} dz dy dx$ 6
- (b) Find the area bounded between the parabola $x^2 = 4ay$ and $x^2 = -4a(y-2a)$ 6

[TURN OVER

- (c) Solve by the method of variation of parameters

8

$$\frac{d^2y}{dx^2} + y = \sec x \tan x$$

4. (a) Find the length of the cardioid
- $r = a(1 - \cos \theta)$
- lying outside the circle
- $r = a \cos \theta$

6

$$(b) \text{ Solve } \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = 2xe^{3x} + 3e^x \cos 2x$$

6

- (c) Using R.K. Method of fourth order, solve.

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \text{ given } y(0) = 1 \text{ at } x = 0.2, 0.4$$

8

5. (a) Solve
- $x \sin x \, dy + (xycos x - y \sin x - 2)dx = 0$

$$(b) \text{ Solve } \frac{dy}{dx} = 2 + \sqrt{xy} \text{ with } x_0 = 1.2, y_0 = 1.6403 \text{ by modified Euler's method, for } x = 1.4 \text{ correct to 4-decimal places, (taking } h = 0.2)$$

6

6

- (c) Evaluate
- $\int_0^6 x f(x) dx$
- by

(a) Trapezoidal rule

(b) Simpson's 1/3rd rule

using the following table

8

x	0	1	2	3	4	5	6
f(x)	0.146	0.161	0.176	0.190	0.204	0.217	0.230

6. (a) The charge Q on the plate of a condenser of Capacity C charged through a resistance R by a steady voltage V satisfies the differential equation

6

$$R \frac{dQ}{dt} + \frac{Q}{C} = V, \text{ If } Q = 0 \text{ at } t = 0, \text{ show that } i = \frac{V}{R} e^{-\frac{t}{RC}} \therefore i = \frac{dQ}{dt}$$

- (b) Evaluate
- $\iint_A x^2 dx dy$
- where A is the region in the first quadrant bounded by the hyperbola
- $xy = 16$
- and the lines
- $y = x, y = 0$
- and
- $x = 8$
- .

6

- (c) Find the volume of the tetrahedron bounded by the planes,
- $x = 0, y = 0, z = 0$
- and
- $x + y + z = a$

8